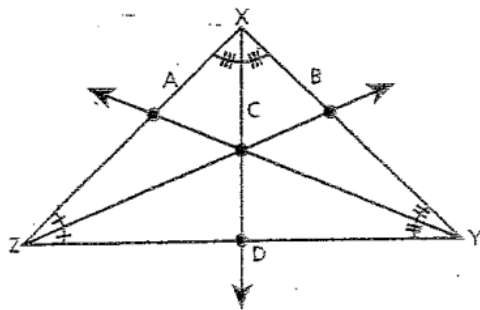# The Angle Bisectors of a Triangle

The **angle bisector** is a line, segment, or ray that bisects the angle of a triangle. Every triangle has three angle bisectors. An angle bisector does not necessarily bisect the opposite side.
$\overrightarrow{ZB}$, $\overrightarrow{XD}$, and $\overrightarrow{YA}$ are angle bisectors.

The common point of all three is the **incenter**. The distance is the same from each side to the incenter. Point C is the incenter.

If $\angle XZD = 44°$, then bisector $\angle CZD = 22°$.

Use the diagrams to answer the questions.
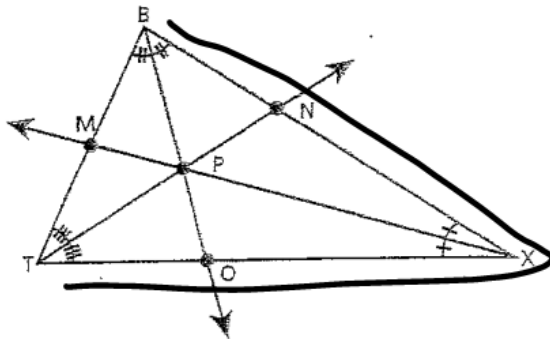
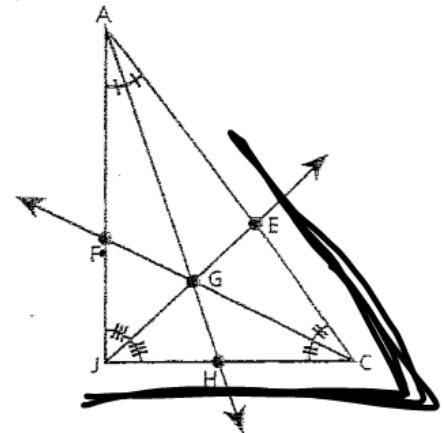**1** Name the angle bisectors.

$\overrightarrow{TN}$ $\qquad$ $\overrightarrow{OB}$ $\qquad$ $\overrightarrow{XM}$

2 What is the incenter of the triangle?

P

3 If $\angle BTX = 68°$, name its two bisector angles and their measures.

$\angle BTN = 34°, \angle NTX = 34°$

4 If $\angle BXT = 32°$, name its two bisector angles and their measures.
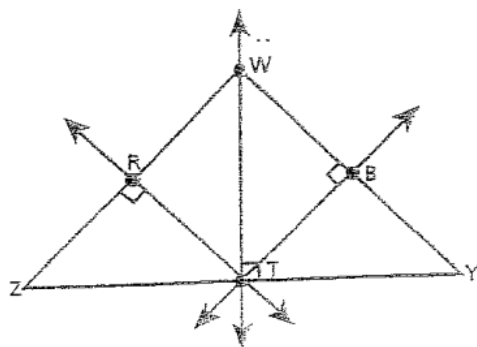
$\angle MXT = 16°, \angle MXB = 16°$

5 Name the angle bisectors.

$\overrightarrow{CF}$ $\qquad$ $\overrightarrow{JE}$ $\qquad$ $\overrightarrow{AH}$

6 What is the incenter of the triangle?

G

7 If $\angle ACJ = 35°$, name its two bisector angles and their measures.

$\angle FCA = 17.5°, \angle FCJ = 17.5°$

8 If $\angle JCA = 55°$, name its two bisector angles and their measures.

$\angle ACF = 27.5°, \angle FCJ = 27.5°$

The Perpendicular Bisectors of a Triangle

The perpendicular bisector is a line, segment, or ray that bisects one side and is perpendicular to it. Every triangle has three perpendicular bisectors. $\overleftrightarrow{RT}$, $\overleftrightarrow{WT}$, and $\overleftrightarrow{BT}$ are perpendicular bisectors.

The common point of all three is the circumcenter. It is the same distance from each vertex of the triangle.
Point T is the circumcenter.
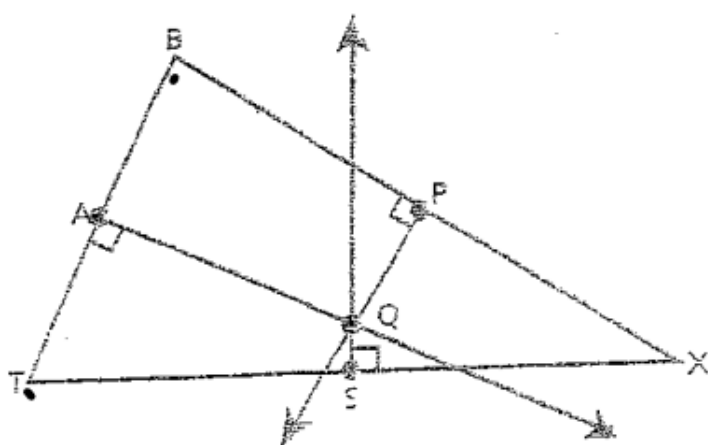
① Name the perpendicular bisectors.

$\overrightarrow{AQ}$ $\overrightarrow{SQ}$ $\overrightarrow{PQ}$
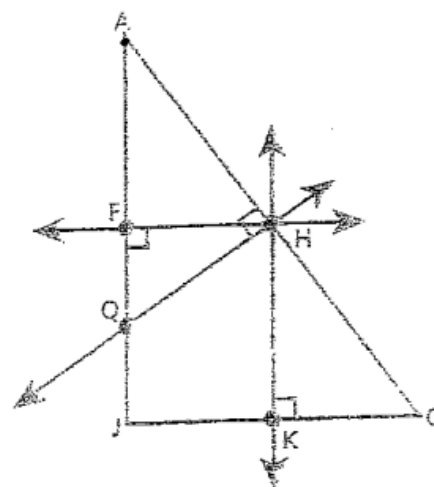
② What is the circumcenter of the triangle?

Q

③ Is $\overline{BP}$ part of a perpendicular bisector or a side?

side

④ If segments are drawn from B to Q and from T to Q, are the segments congruent?

yes

⑤ Name the perpendicular bisectors.

$\overleftrightarrow{HK}$ $\overleftrightarrow{HF}$ $\overleftrightarrow{HQ}$

⑥ What is the circumcenter of the triangle?

H

⑦ Are $\overline{AH}$ and $\overline{HC}$ congruent?

yes

⑧ If a segment is drawn from J to H, is it congruent to $\overline{CH}$?
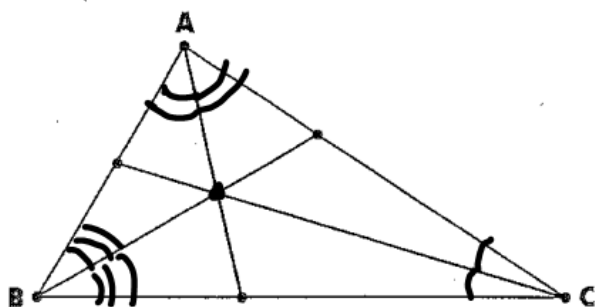
yes

Angle Bisector and Perpendicular Bisector Summary

There are __3__, ∠ bisectors, and ⊥ bisectors in __one__ triangle.

They intersect in __one__ point. Sometimes that point is __inside__ sometimes __outside__, and sometimes __on__ the triangle depending on the segment & triangle.

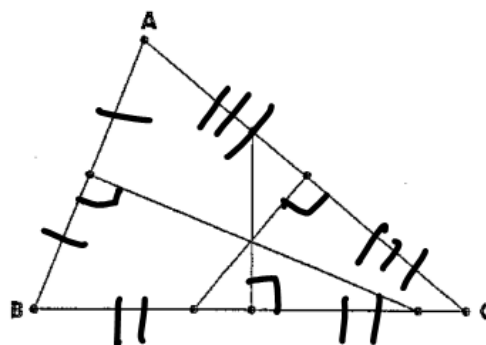### Angle Bisector (p.274)

Bisects an __angle__

Label the picture to indicate that each segment is an **angle bisector**

What happened to the intersection as the triangle changed?

⤷move **within** the triangle

Perpendicular Bisector (p.272)

Passes thru a **midpoint** and is __⊥__ to a side

Label the picture to indicate that each segment is a **perpendicular bisector**

What happened to the intersection as the triangle changed?

⤷moves depending on the kind of △ created

Sometimes, Always or Never

1. A perpendicular bisector of a triangle passes through the midpoint of a side of the triangle. **always**
2. The angle bisectors of a triangle intersect at a single point. **always**
3. The angle bisectors of a triangle meet at a point outside the triangle. **never**

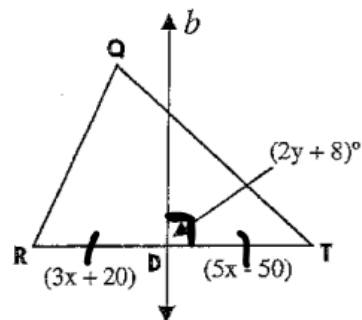**Ex 1**   **Given:** $\overline{TY}$ is an angle bisector
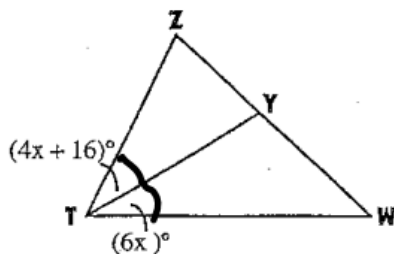
What does the given tell you?

$$\angle ZTY \cong \angle YTW$$

Label this in your picture.

Solve for missing variable(s).

$$4x + 16 = 6x$$
$$16 = 2x$$
$$x = 8$$

Ex 2   **Given:** Line b is a $\perp$ bisector

What does the given tell you?

$$2y + 8 = 90°, \ \overline{RD} \cong \overline{DT}$$

Label this in your picture.

Solve for missing variables.

$$2y + 8 = 90 \qquad 3x + 20 = 5x - 50$$
$$2y = 82 \qquad\qquad 70 = 2x$$
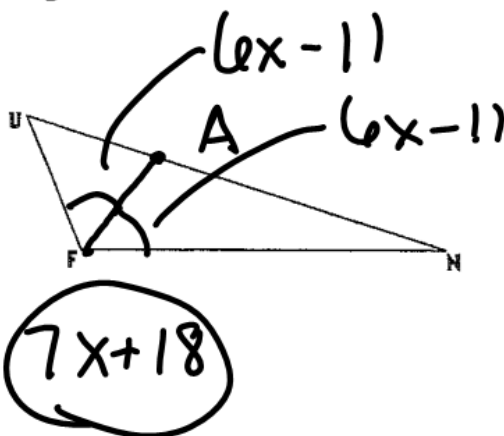$$y = 41 \qquad\qquad 35 = x$$

Ex 3   **Given:** $\overline{FA}$ is the angle bisector of $\angle UFN$. <u>Sketch</u> $\overline{FA}$ in the triangle below.
     $m\angle UFN = (7x + 18)°$ and $m\angle NFA = (6x - 11)°$.

What does the given tell you?

Label this in your picture.

Solve for missing variable(s).

$$7x + 18 = 12x - 22$$
$$40 = 5x$$
$$x = 8$$